Acceptable Programs Revisited

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Slides for presentation of the paper with the same title.

The full paper will be published in the proceedings.

Acceptable Programs

Apt & Pedreschi 1993

P normal logic program.

p, q predicate symbols occurring in P.

- p *refers to* q if there is a clause in P with p in its head and q in its body.
- p *depends on* q if (p, q) is in the reflexive, transitive closure of *refers to*.
- Neg_P: set of pred. symbols in P occurring in a negative literal in the body of a clause in P.
- $N := \text{Neg}^*_P$: set of pred. symbols in P on which the predicate symbols in Neg_P depend.
- P^-: set of clauses in P containing a predicate symbol from N in the head.

$l : B_P \to \mathbb{N}$ level mapping.

I model for P and supported model of P^-.

P is *acceptable* (wrt. I and l) if $(*)$ holds:

$(*)$ For each clause $A \leftarrow L_1, \ldots, L_n$ in ground(P) and all $i = 1, \ldots, n$

$$\text{if } I \models \bigwedge_{j=1}^{i-1} L_j, \text{ then } l(A) > l(L_i).$$

Issues Concerning Preinterpretations

Fitting 1994: Model need not be Herbrand.

Marchiori 1996: If program completion is consistent and underlying language contains infinitely many constant and function symbols then acceptable programs are exactly the programs terminating under Chan's (Chan 1988) constructive negation.

Conclude: Choice of underlying language and preinterpretation is important!

Examples

$$\begin{array}{rcl} r(0) & \leftarrow & \neg p(0), \neg r(0) \\ p(0) & \leftarrow & \neg q(X) \\ q(0) & \leftarrow & \end{array}$$

Acceptable wrt. $M = \{p(0), q(0)\}$, language with constants $\{0, 1\}$.
Not acceptable wrt. language with constants $\{0\}$.

$$\begin{aligned} r(0) &\leftarrow \neg q(X), \neg r(0) \\ q(0) &\leftarrow \end{aligned}$$

Acceptable wrt. $M = \{q(0)\}$, language $\{0\}$.
Not acceptable wrt. language $\{0, 1\}$.
If language contains infinitely many constant symbols then comp(P) is not consistent.

$$\begin{aligned} r(0) &\leftarrow \neg q(X), r(0) \\ q(0) &\leftarrow \end{aligned}$$

Acceptable wrt. $M = \{q(0)\}$, language $\{0\}$.
Goal $\leftarrow r(0)$ does not terminate under Chan's constructive negation.

$$\begin{aligned} r(0) &\leftarrow \neg q(X), r(0) \\ q(f(0)) &\leftarrow \end{aligned}$$

Not acceptable under Herbrand preinterpretation (in any language).
Acceptable if domain of preinterpretation is singleton set $\{0\}$ and f identity function on $\{0\}$.

Defining Acceptability

$\mathcal{L}$ (suitable) first order language.
J preinterpretation based on $\mathcal{L}$ with domain $\mathcal{D}$.
$B_{P,J}$ set of all $p(d_1, \dots, d_n)$ where
p predicate symbol in $\mathcal{L}$ and $d_1, \dots, d_n \in \mathcal{D}$.
$p(d_1, \dots, d_n) \in B_{P,J}$ is J-*ground instance* of atom $p(t_1, \dots, t_n)$ over $\mathcal{L}$
if exists J-variable assignment v such that $t_i|v$ is equal to d_i (relative to J) for all i.
$\text{ground}_J(P)$ set of all J-ground instances of clauses in P.

P is called (J-)*acceptable* if
- J preinterpretation over some language $\mathcal{L}$,
- I model of $\text{ground}_J(P)$ (a J-*model*)

st. $I \cap N$ is supported model of $\text{ground}_J(P^-)$,
- $l : B_{P,J} \to \mathbb{N}$ level mapping and
- condition $(*)$ holds wrt. l, I (and $\text{ground}_J(P)$).

$I_{P,J} = 2^{B_{P,J}}$ set of all J-interpretations of P.

Unique Supported Models

Apt & Pedreschi 1993: Acceptable programs have unique supported (Herbrand) models.

Proof uses Fitting's three-valued Kripke-Kleene semantics (1985).

Result carries over to J-acceptable programs. From this we can derive an alternative approach as follows.

The Atomic Topology Q on $I_{P,J}$

Characterization:

A net (I_λ) converges in Q iff
for every J-ground atom A exists index α
st. either for all $\beta > \alpha$ we have $I_\beta \models A$
or for all $\beta > \alpha$ we have $I_\beta \not\models A$.

Limit is set of all $A \in B_{P,J}$
st. exists α with $I_\beta \models A$ for all $\beta > \alpha$.

Q is Cantor topology.

First Main Result

Let P be J-acceptable.
Then the sequence $(T^n_{P,J}(\emptyset))$ converges in the atomic topology to the unique supported J-model $M_{P,J}$ of P.

Proof idea

Apt & Pedreschi 1993:
Fitting's (1985) operator Φ yields minimal model

$$M = \Phi\uparrow\omega$$

in Kleene's strong 3-valued logic. M is total.

For $I_n = T^n_{P,J}(\emptyset)$ and $K_n = \Phi\uparrow n$ we obtain

$$K_n^+ \subseteq I_n \subseteq {}^cK_n^-$$

for all n.

Iterates of $T_{P,J}$ are "squeezed between" iterates of Φ.

Since $\Phi\uparrow\omega$ is total, $T^n_{P,J}(\emptyset)$ converges in Q.

Canonical Level Mappings

Locally Hierarchical Programs

• Program Transformation:

P normal logic program, I J-model of P.

• For each J-ground clause $A \leftarrow L_1, \ldots, L_n$ determine maximal i st. $I \models L_1 \wedge \cdots \wedge L_i$.

Replace clause with

$A \leftarrow L_1, \ldots, L_{i+1}$ if $i \neq n$

$A \leftarrow L_1, \ldots, L_n$ if $i = n$.

• Resulting (J-ground) program is called P_I.

If P acceptable then P_I locally hierarchical (Cavedon 1989/91), even acyclic (Bezem 1989). ie. exists level mapping l st. $l(A) > l(L_i)$ for all i in every J-ground clause $A \leftarrow L_1, \ldots, L_n$.

Seda & Hitzler 1997: loc. hierarchical program P has pointwise minimal level mapping l'_P (*canonical level mapping*).

l'_P can be obtained by inductive construction which succeeds iff P is locally hierarchical, and is not transfinite iff P is acyclic.

Acceptable Programs

P J-acceptable (wrt. I and l). Then:
(i) Obtain P_I (Program Transformation).
(ii) Obtain l'_{P_I} (as for acyclic programs).
(iii) Set $l_P(A) = l'_{P_I}(A)$ for A occurring in P_I,
$l_P(A) = 0$ otherwise.

Remarks and Results

If Step (ii) does not succeed (or is transfinite),
then P is not J-acceptable.

If P is J-acceptable,
then P is J-acceptable wrt. I and l_P.

If P is J-acceptable wrt. some I and l,
then $l_P(A) \leq l(A)$ for all $A \in B_{P,J}$.

Further Results

Let P be J-acceptable wrt. some I and some l.

- P is J-acceptable wrt. $M_{P,J}$.

Let l_P be obtained using the model $M_{P,J}$.

Second Main Result
P is J-acceptable wrt. $M_{P,J}$ and l_P.

- $l_P(A) \leq l(A)$ for all $A \in B_{P,J}$.

- $M_{P,J} = \bigcap \mathcal{M}$
where $\mathcal{M}$ is the set of all J-models
with respect to which P is J-acceptable.

- There is only one minimal model with respect to which P is J-acceptable.

Characterizing Acceptability

Main Result

Let P be a normal logic program.

Then P is J-acceptable if and only if:

(i) $T^n_{P,J}(\emptyset)$ converges in Q to some J-model $M (= M_{P,J})$.

(ii) l_P can be constructed from $M_{P,J}$.

(iii) P satisfies $(*)$ from the definition of acceptability with respect to l_P and $M_{P,J}$.

Corollary

A normal logic program P is terminating under Chan's constructive negation iff it satisfies (i), (ii), (iii) above for some preinterpretation J whose domain contains infinitely many constants and functions.