Default reasoning over domains and concept hierarchies

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Contents

- A motivation: Semantic Web
- A prototype of a generic reasoning paradigm
  - — over hierarchical knowledge
  - — in the tradition of answer set programming

. . . work in progress . . .

Motivation

Semantic Web Challenge: making the internet machine-usable.

Conceptual knowledge extracted from/provided with web pages.
$\rightsquigarrow$ Ontologies.

Some current efforts:
How to do **rule-based reasoning** over conceptual knowledge.
How to do **non-monotonic reasoning** over conceptual knowledge.

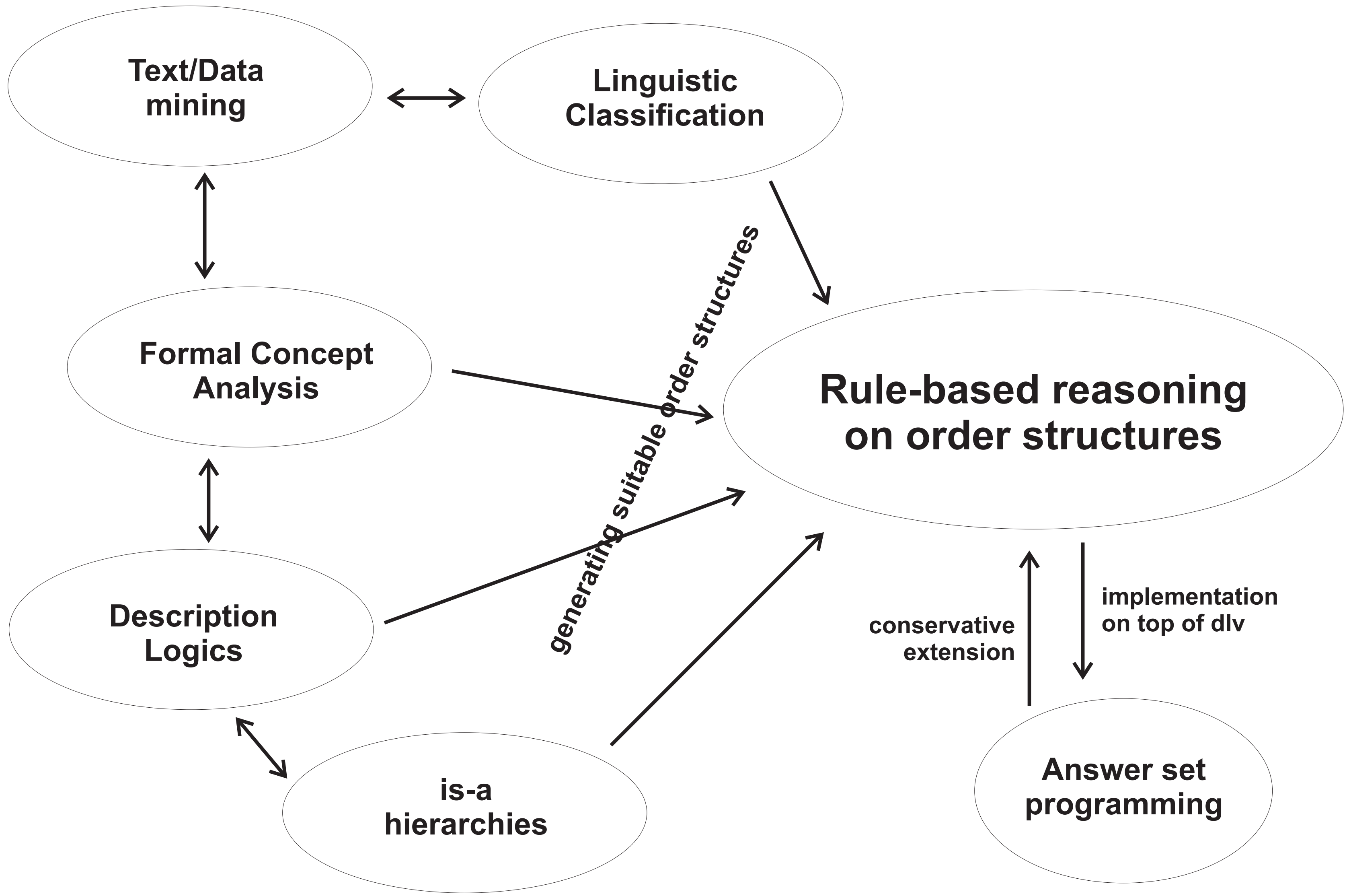
Towards a reasoning system

Seek:

- Generic
- rule-based
- non-monotonic reasoning system
- covering different kinds of conceptual structures,
- in the tradition of answer set programming (ASP).

Conceptual knowledge is **hierarchical**

$\leadsto$ Reasoning on **order structures**.


Text/Data mining
Linguistic Classification
Formal Concept Analysis
Rule-based reasoning on order structures
generating suitable order structures
Description Logics
is-a hierarchies
conservative extension
implementation on top of dlv
Answer set programming

Logic RZ on order structures

(Rounds & Zhang IC 2001)

$(D, \sqsubseteq)$: coherent algebraic cpo (e.g. finite)

Clause: $X \subseteq \mathsf{K}(D)$ finite (disjunction)

$w \in D$, then $w \models X$ iff $\exists x \in X. x \sqsubseteq w$

Theory T: set of clauses (conjunction)

$w \models T$ iff $\forall X \in T. w \models X$

$T \models X$ iff $\forall w \in D. w \models T \implies w \models X$

- compact; comes with proof theory
- *domain logic*: for characterizing Smyth powerdomains

Logic RZ

Proof theory: (H JEEEC 2004)

$$\frac{X; \qquad a \in X; \qquad y \sqsubseteq a}{\{y\} \cup (X \setminus \{a\})}$$

$$\frac{X; \qquad y \in \mathsf{K}(D)}{\{y\} \cup X}$$

$$\frac{X_1 \quad X_2; \qquad a_1 \in X_1 \quad a_2 \in X_2}{\mathrm{mub}\{a_1, a_2\} \cup (X_1 \setminus \{a_1\}) \cup (X_2 \setminus \{a_2\})}$$

Logic RZ is compact.

Formal Concept Analysis (FCA)

A mathematical methodology for obtaining concept hierarchies from object-attribute relationships.

Used in data- and textmining, ontology creation, symbolic data analysis.

Applications in software engineering, theoretical computer science, social network analysis, psychology, biology, . . .

qualitative cluster analysis

Formal Concept Analysis (FCA)

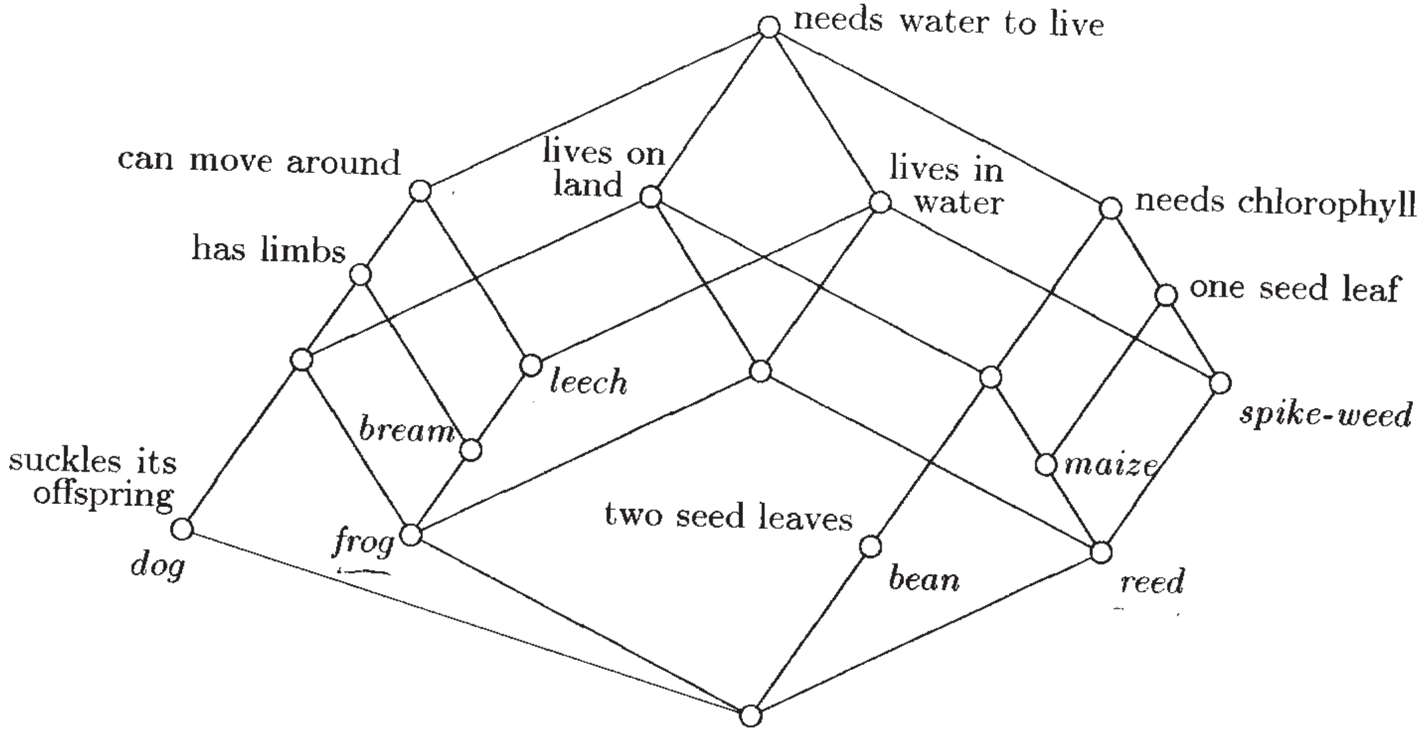


Figure 1.4 Concept lattice for the educational film "Living beings and water".

(picture: Ganter & Wille, Formal Concept Analysis, Springer, 1999.)

FCA and Logic RZ

(H & Wendt ICCS 2003; H 2004)

Theorem

$(D, \sqsubseteq)$ coherent algebraic cpo.

$(L, \leq)$ AOC of a formal context (G, I, M).

$\iota : L \rightarrow D$ order-reversing injection
with $\mathsf{K}(D) \subseteq \iota(L)$.

$B = \{m_1, \ldots, m_n\} \subseteq M$ mit $\iota(m_i) \in \mathsf{K}(D)$ for all i.

Then

$$B'' = \{m \in M \mid \{\{\iota(m_1)\}, \ldots, \{\iota(m_n)\}\} \models \{\iota(m)\}\}.$$

Concept closure is the conjunctive fragment of the logic RZ.

RZ logic programming

(Rounds & Zhang IC 2001)

Add material implication: $X \leftarrow Y$ for X, Y clauses.

$w \models P$: if $w \models Y$ for $X \leftarrow Y \in P$, then $w \models X$.

Propagation rule $\mathrm{CP}(P)$:

$$\frac{X_1 \quad \dots \quad X_n; \quad a_i \in X_i; \quad Y \leftarrow Z \in P; \quad \mathrm{mub}\{a_1, \dots, a_n\} \models Z}{Y \cup \bigcup_{i=1}^{n}(X_i \setminus \{a_i\})}$$

Semantic operator on theories:

$$\mathcal{T}_P(T) = \mathrm{cons}\left(\{Y \mid Y \text{ is a } \mathrm{CP}(P)\text{-consequence of } T\}\right).$$

- ▶ $\mathcal{T}_P$ is Scott continuous.
- ▶ $\mathrm{fix}\left(\mathcal{T}_P\right) = \mathrm{cons}(P)$.

Addition of default negation

(H 2004)

Allow rules of the form $X \leftarrow Y, \sim Z$, where X, Y, Z are clauses.

P program consisting of such rules, $w \in D$. Define P/w:

Replace $Y, \sim Z$ by Y if $w \not\models Z$.

Remove rule if $w \models Z$.

w *min-answer model* for P if w is minimal with $w \models \mathrm{fix}(\mathcal{T}_{P/w})$.

Theorem

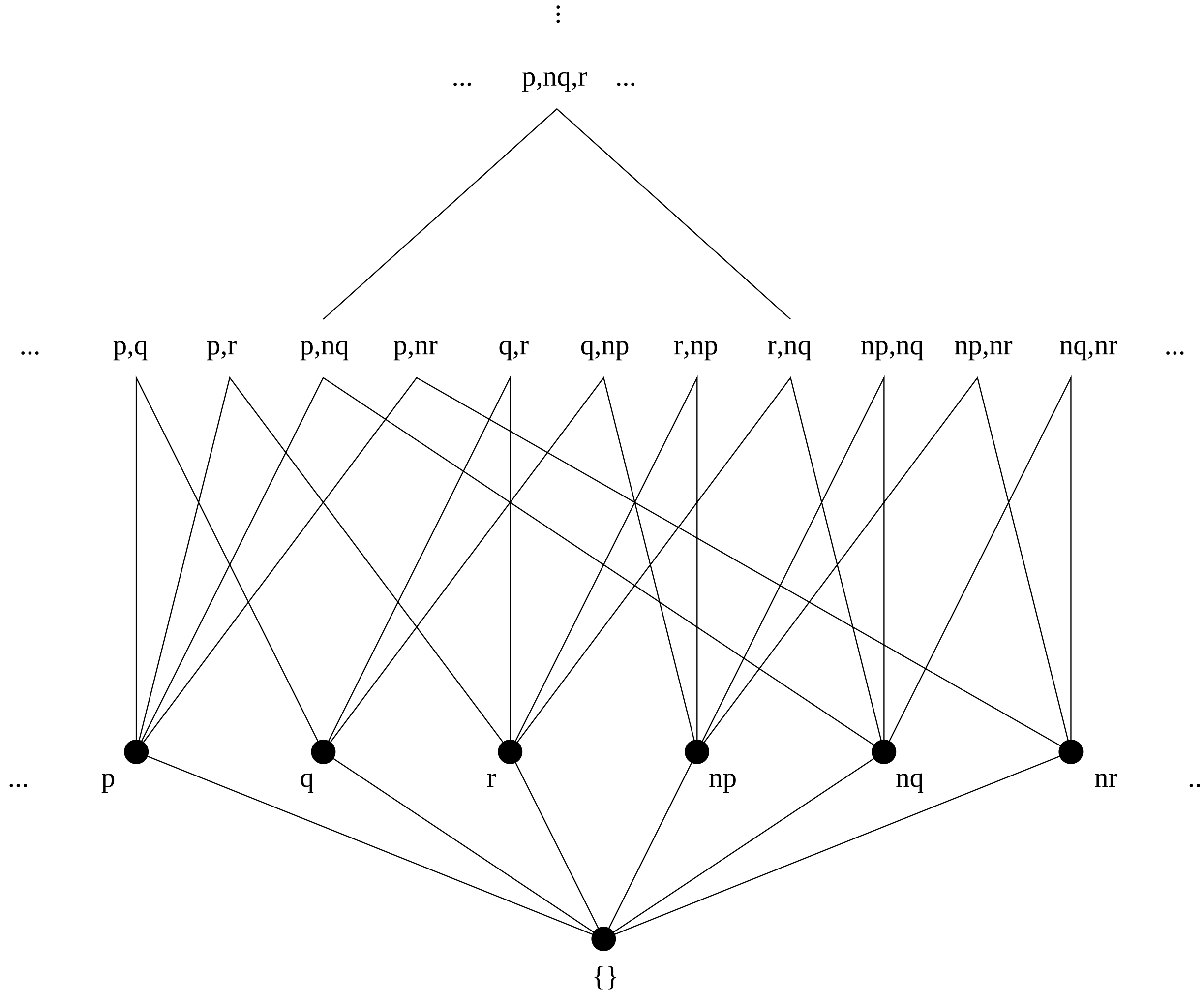
Answer set programming

with extended disjunctive programs (e.g. dlv-system)

is exactly

RZ logic programming with $D = \mathbb{T}^\omega$.

Plotkin's $\mathbb{T}^\omega$



$X \leftarrow Y, \sim Z$

Outlook

Idea:

Conceptual knowledge provided in hierarchical form.

Rule-based reasoning over this knowledge.

Conceptual Knowledge:

Ontologies.

Attribute exploration over T-boxes.

FCA-textmining.

Other methods (linguistics?).

Rule-Based Reasoning:

Conceptual knowledge implicit.

In the sense of mainstream ASP.

Implementation e.g. on top of dlv.

Formal Concept Analysis (FCA)

(FCA: an approach to data mining and analysis; Ganter & Wille 1999)

G set of objects; M set of attributes. $C \subseteq G \times M$ *formal context*.

$A \subseteq G$ then $A' = \{m \in M \mid (\forall g \in A)(g, m) \in C\}$.
$B \subseteq M$ then $B' = \{g \in G \mid (\forall m \in B)(g, m) \in C\}$.

Formal concept: Pair (A, B) with $A' = B$, $A = B'$.
Equivalently: All (B', B'') for $B \subseteq M$.

Formal concept lattice:
complete lattice of all concepts ordered by $\supseteq$ in second argument.

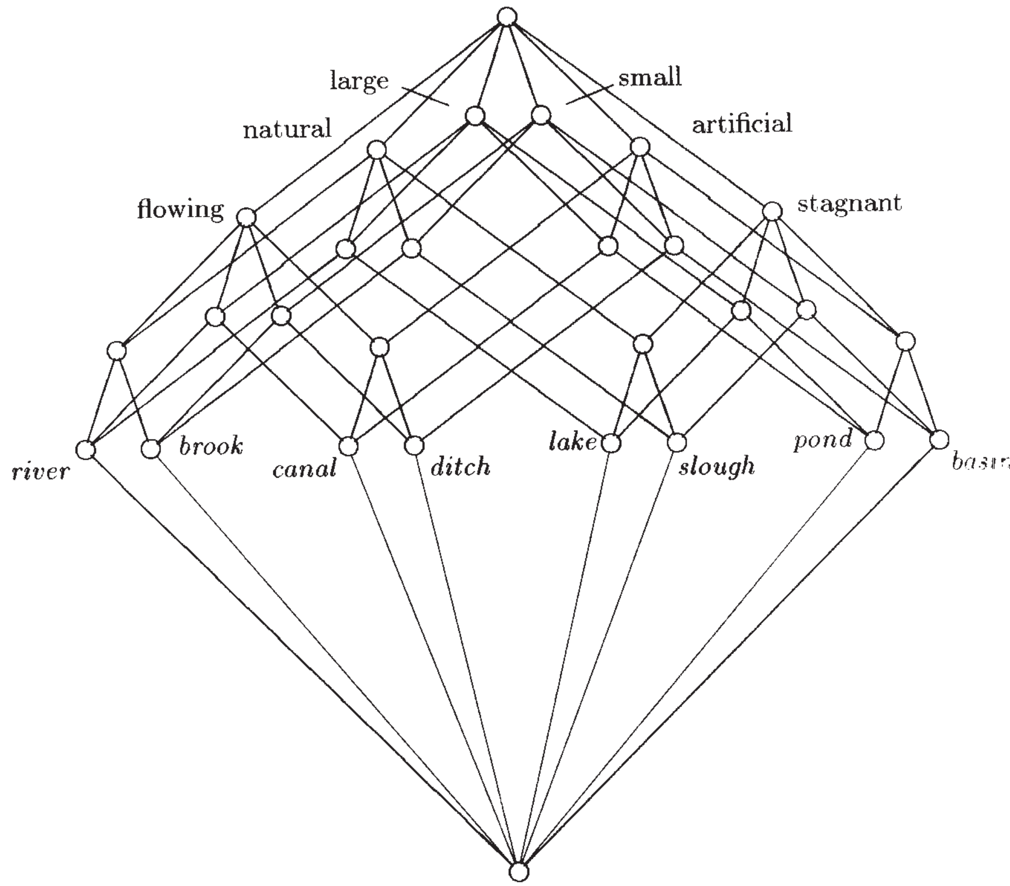


Figure 2.9 An additive line diagram of the concept lattice of a *lexical field "waters"*. The set representation is based on the irreducible attributes, i.e. the positioning of the attribute concepts determines that of all remaining concepts. If we interpret the line segments between the unit element and the attribute concepts as vectors. we obtain the position of an arbitrary concept by the sum of the vectors belonging to attributes of its concept intent starting from the unit element. Other diagrams for the same lattice can be found in Figure 2.10.

Domain theory: coherent algebraic cpos

cpo: directed complete partial order with bottom $(D, \sqsubseteq)$

$c \in \mathsf{K}(D)$ (compact) iff $(\forall A$ directed$)(d \sqsubseteq \bigsqcup A \implies (\exists a \in A) d \sqsubseteq a)$

cpo *algebraic*: $(\forall x)\,(x = \bigsqcup(x{\downarrow} \cap \mathsf{K}(D)))$

Scott topology: base $\{\uparrow c \mid c \in \mathsf{K}(D)\}$

coherent: finite intersections of compact-opens are compact-open

Examples: Finite posets with bottom. Powersets. $\mathbb{T}^\omega$.

Logic RZ and Formal Concept Analysis (FCA)

(H & Wendt ICCS 2003)

Consider subposet D of all $(\{b\}', \{b\}'')$, $b \in M$,
and all $(\{a\}'', \{a\}')$, $a \in G$, ordered reversely (add $\perp$).

If D is finite, then

for $D \supseteq \{b_i \mid i \in I\} = B \subseteq M$ we have

$$B'' = \{b \in M \mid \{\{b_i\} \mid i \in I\} \models \{b\}\}.$$

Extension of answer set programming

Consider $\mathbb{T}^\omega$.

Consider programs P with rules $X \leftarrow Y, \sim Z$ such that:

X contains only atoms in $\mathbb{T}^\omega$.

Y is a singleton clause.

Z contains only atoms in $\mathbb{T}^\omega$ or $\perp$.

These programs are exactly extended disjunctive programs.

Min-answer models w correspond to *answer sets* $\{L \text{ atom} \mid w \models \{L\}\}$ and vice-versa.