

McCulloch-Pitts Networks

- ▶ McCulloch & Pitts: A Logical Calculus and the Ideas Immanent in the Nervous Activity. *Bulletin of Mathematical Biophysics* 5, 115-133 (1943).
 - ▷ Is there a logical calculus modelling the activities of the nervous system?
- ▶ McCulloch-Pitts networks:
 - ▷ U set of logical threshold units.
 - ▷ $W \subseteq U \times U$ set of weighted connections.
 - ▷ $w_{ij} = 0$ iff no connection from u_j to u_i .
 - ▷ Discrete time $t = 1, 2, \dots$
 - ▷ Synchronous updates.

McCulloch-Pitts Networks: Examples

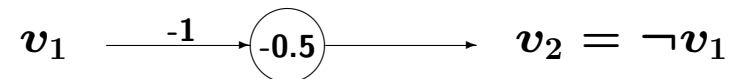
▶ Disjunction:



▶ Conjunction:



▶ Negation:



▶ **Observation:** Each propositional logic formula can be represented by a network of binary threshold units.

McCulloch-Pitts Networks: Input and Output Units

- ▶ **Input units:** $U_I = \{u_i \in U \mid (\forall u_j \in U) w_{ij} = 0\}$
- ▶ **Output units:** $U_O = \{u_i \in U \mid (\forall u_j \in U) w_{ji} = 0\}$
- ▶ **Schematic view of a McCulloch-Pitts network with m input and n output units:**



- ▶ **At each point in time:**
 - ▶ Network receives input.
 - ▶ Computes internal state.
 - ▶ Produces output value.

Finite Automata

- ▶ A finite automaton consists of:
 - ▶ Σ , a finite set of input symbols,
 - ▶ Φ , a finite set of output symbols,
 - ▶ Q , a finite set of states,
 - ▶ $q_0 \in Q$, an initial state,
 - ▶ $F \subset Q$, a set of final states
 - ▶ $\delta : Q \times \Sigma \rightarrow Q$, a state transition function,
 - ▶ $\rho : Q \rightarrow \Phi$, an output function.

- ▶ **Exercise:** Consider the following finite automaton:

$$\Sigma = \Phi = \{0, 1\}, Q = F = \{p, q, r\}, q_0 = p,$$

| | | | |
|--------|-----|-----|-----|
| ρ | p | q | r |
| | 0 | 0 | 1 |

| | | |
|----------|-----|-----|
| δ | 0 | 1 |
| p | q | p |
| q | r | q |
| r | r | r |

What is computed by this automaton?

McCulloch-Pitts Networks and Finite Automata

- ▶ **Arbib:** Brains, Machines and Mathematics. Springer, 2nd edition (1987).
- ▶ **Exercise:** Show that each McCulloch-Pitts network is a finite automaton.
- ▶ **Theorem:** Each finite automaton is a McCulloch-Pitts network.
- ▶ **Exercise:** Construct the McCulloch-Pitts network modelling the finite automaton specified previously.
- ▶ **McCulloch-Pitts networks are not just simple reactive systems, but their behavior depends on previous inputs as well as the activity within the network.**
 - ▶ **Example:**

