McCulloch-Pitts Networks

- McCulloch & Pitts: A Logical Calculus and the Ideas Immanent in the Nervous Activity. Bulletin of Mathematical Biophysics 5, 115-133 (1943).
  - Is there a logical calculus modelling the activities of the nervous system?

- McCulloch-Pitts networks:
  - U set of logical threshold units.
  - $W \subseteq U \times U$ set of weighted connections.
  - $w_{ij} = 0$ iff no connection from u_j to u_i.
  - Discrete time $t = 1, 2, \ldots$.
  - Synchronous updates.

McCulloch-Pitts Networks: Examples

- **Disjunction:**

- **Conjunction:**

- **Negation:**

- **Observation: Each propositional logic formula can be represented by a network of binary threshold units.**

McCulloch-Pitts Networks: Input and Output Units

- **Input units:** $U_I = \{u_i \in U \mid (\forall u_j \in U)\, w_{ij} = 0\}$
- **Output units:** $U_O = \{u_i \in U \mid (\forall u_j \in U)\, w_{ji} = 0\}$
- **Schematic view of a McCulloch-Pitts network with m input and n output units:**

- **At each point in time:**
  - **Network receives input.**
  - **Computes internal state.**
  - **Produces output value.**

Finite Automata

- A finite automaton consists of:
  - Σ, a finite set of input symbols,
  - Φ, a finite set of output symbos,
  - Q, a finite set of states,
  - $q_0 \in Q$, an initial state,
  - $F \subset Q$, a set of final states
  - $\delta : Q \times \Sigma \longrightarrow Q$, a state transition function,
  - $\rho : Q \longrightarrow \Phi$, an output function.

- Exercise: Consider the following finite automaton:

$$\Sigma = \Phi = \{0, 1\},\ Q = F = \{p, q, r\},\ q_0 = p,$$

ρ	p	q	r
	0	0	1

,

δ	0	1
p	q	p
q	r	q
r	r	r

.

What is computed by this automaton?

McCulloch-Pitts Networks and Finite Automata

- **Arbib: Brains, Machines and Mathematics. Springer, 2nd edition (1987).**
- **Exercise: Show that each McCulloch-Pitts network is a finite automaton.**
- **Theorem: Each finite automaton is a McCulloch-Pitts network.**
- **Exercise: Construct the McCulloch-Pitts network modelling the finite automaton specified previously.**
- **McCulloch-Pitts networks are not just simple reactive systems, but their behavior depends on previous inputs as well as the activity within the network.**
  - ▷ **Example:**